Coordinating Pricing and Supply of Public Interest Goods Using Government Rebates and Subsidies

Anton Ovchinnikov
Queen’s School of Business
143 Union Str West, Kingston, ON, K7L3N6, Canada
anton.ovchinnikov@queensu.ca

Gal Raz
Darden School of Business, University of Virginia
100 Darden Boulevard, Charlottesville, VA, 22903, USA
razg@darden.virginia.edu

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Abstract

This paper presents a stylized framework based on the newsvendor model with pricing for analyzing the design of government incentives for public interest goods (goods with externalities such as electric vehicles.) We extend the newsvendor model with pricing to account for the consumption externality inherent in public interest goods and analyze the government’s ability to coordinate their pricing and supply through the use of rebates and subsidies. Our model allows for goods with both positive and negative externalities, and considers three government intervention mechanisms: the joint mechanism that uses both subsidies and rebates, and two simplified mechanisms which use only rebates or only subsidies. The goal of the intervention is to coordinate the system in order to achieve the maximal welfare which in our model consists of the firm’s profit, consumer surplus, and externality benefit net the government cost.

We find that the joint mechanism coordinates the system, but results in a negative subsidy (i.e., a tax) unless the externality is very small. The simplified mechanisms mostly result in positive rebates and subsidies, but generally do not coordinate the system. We show that when the government uses the subsidy-only mechanism the welfare losses can be large, but when it uses the rebates-only mechanism the welfare losses are in most cases very small, making the rebates-only mechanism a reasonable policy alternative.

We finally apply our model to the case of Chevy Volt, a leading electric vehicle (EV) in North America manufactured by General Motors. We estimate all model parameters from industry data and present a comprehensive numerical study in which we compare the current rebate-only incentive with those suggested by our model. An important qualitative finding is that while the current incentive is structurally suboptimal, the resultant welfare loss is very small.
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Statement of Managerial Relevance

To meet President Obama’s goal to become the first country in the world to have one million electric vehicles (EVs) on the road by 2015, the U.S. government had been subsidizing their production and providing a $7,500 tax credits (rebates) to consumers who were purchasing them. This paper uses a stylized framework based on the newsvendor model with pricing for analyzing the design of such government subsidies and rebates. Our model considers the joint mechanism that uses both subsidies and rebates, and two simplified mechanisms which use only rebates or only subsidies. We show that the joint mechanism coordinates the system (i.e., results in no welfare losses), but often with a negative subsidy (i.e., a tax). The subsidy-only mechanism leads to large welfare losses, but under the rebates-only mechanism the welfare losses are in most cases very small. We apply our model to the case of Chevy Volt, a leading EV in North America manufactured by GM. Estimating model parameters from industry data we present a comprehensive numerical study to compare the current rebate-only incentive with those suggested by our model. We find that while the current incentive is structurally suboptimal, the resultant welfare loss is very small.
1 Introduction

On January 25, 2011, President Obama stated that the United States would become the first country in the world to have one million electric vehicles (EVs) on the road by 2015\textsuperscript{1}. The President’s statement was reiterating a goal he previously announced as a presidential candidate in 2008\textsuperscript{2} and as a result of it, starting from 2009, the U.S. government had been subsidizing the production of EVs and providing a $7,500 tax credits to consumers who were purchasing them\textsuperscript{3}. The motivation behind the government support for EVs is the fact that they are considered \textit{public interest goods}\textsuperscript{4}. What differentiates such goods from other goods is the effect of \textit{externality} – a situation where the production and consumption of the good imposes indirect involuntary benefits or costs on other economic agents who are outside the market place for that good. For example, EVs, such as Chevy Volt, Nissan Leaf or Tesla Model S, reduce carbon emissions compared to traditional vehicles thus affecting drivers of those vehicles, drivers of other vehicles, and even those who do not drive at all. Other examples of public interest goods include desirable products such as medical goods (e.g., vaccines), emergency supplies (e.g., power generators), energy-efficient appliances (e.g., water-saving toilets) and eco-consumables (e.g., organic fertilizer as opposed to synthetic), as well as undesirable products, such as unhealthy foods (e.g., cigarettes) and pollutants.

Because externality has an indirect effect, its societal value (or cost) is factored neither in the profit of the firm that produces the good nor in the utility of consumers who purchase the good. Therefore the pricing and supply processes for public interest goods are often subject to government interventions. Such interventions include payments to consumers, to which we refer as \textit{rebates}, and payments to firms, to which we refer as \textit{subsidies}. Designing such interventions requires trading-off various components of social welfare (firms’ profits,

\textsuperscript{1}: http://www.whitehouse.gov/the-press-office/2011/01/25/remarks-president-state-union-address
\textsuperscript{2}Presidential candidate speech in Lansing, Michigan on August 4, 2008 and then he first reiterated the goal as President at a speech in Pomona, California on March 19, 2009
\textsuperscript{3}http://www.irs.gov/Businesses/Plug-In-Electric-Vehicle-Credit-(IRC-30-and-IRC-30D).
\textsuperscript{4}Not to be confused with \textit{public goods}. Defined as the non-rival, non-excludable goods, such as broadcast television or national defense, public goods fundamentally misfit the newsvendor logic that a certain quantity is produced.
consumer surplus, externality benefit/cost and government costs), and is further complicated by the fact that consumer demand is often uncertain, while manufacturing requires large upfront investments in capacity.

The goal of this paper is to analyze how product externality and demand uncertainty affect the optimal design of rebates and subsidies interventions (a problem to which we refer as the pricing and supply coordination) for public interest goods. We extend the price-setting newsvendor model to account for product externality and apply our model to the problem of determining rebates and subsidies for EVs.

Our paper builds on two streams of literature: the operations management literature (and the price-setting newsvendor model in particular) provides frameworks for analyzing demand uncertainty, while the economics literature provides frameworks for analyzing welfare and externalities. Combining the two, we consider a system that consists of a newsvendor firm that manufactures and sells a public interest good, a population of consumers who buy the good, a general population that experiences externality benefit (or cost) and a government that coordinates/regulates the system. We formulate what we believe is the first model of a price-setting newsvendor that responds to government interventions, solve the model for the benchmark cases of centralized (coordinated or “first-best”) and decentralized (uncoordinated) systems, and then study the ability of rebates and subsidies to coordinate social welfare. With these two tools we consider three government intervention mechanisms: the joint mechanism where the government is using both rebates and subsidies together, and two simplified mechanisms (such as the examples presented above) where the government uses only rebates or only subsidies.

Our modelling framework is stylized, yet quite general. Specifically, we allow externality to be either positive or negative (thus applying to public interests goods that have a negative externality such as cigarettes for which the government taxes consumers, i.e., applies, in our terminology, a negative rebate), and allow rebates and subsidies to be either positive (as per their typical interpretation) or negative (when they can be interpreted as taxes on
consumers or producers). In total we integrate eight government intervention tools in a single parsimonious framework.

Within this framework we generate the following analytical results. First, we show that both uncertainty and externality decrease the optimal price in the centralized solution. For goods with positive or small negative externality this implies that the centralized price is below cost, which reinforces the need for regulation and points to the unattainability of the centralized solution by an unregulated firm or a competitive industry. Second, we show that the joint mechanism coordinates the system (resulting in the same welfare as the centralized solution), while the simplified mechanisms generally do not. But very surprisingly, – and this is our main analytical result, unless externality is negative and large in the absolute value, the optimal joint subsidy is negative, i.e., it is actually a tax. In such cases the government coordinates the system by providing a positive rebate to consumers and then taxing the firm with a negative “subsidy”. The reason for why this approach works is that it is an efficient way to increase service level: the rebate increases effective product margin, in response to which the newsvendor firm sets a higher capacity service level thus producing and selling more units, which are then taxed back to decrease the net government cost. The optimal rebate and subsidy are generally positive under the simplified mechanisms.

Our numerical study is based on empirical data for the example of Chevy Volt, a leading EV in North America. We estimate model parameters based on the Volt’s technical and market data and present two sets of numerical results. First, in the base-case we discuss the implications of our results to the design of government incentives for EVs. Specifically, we show that the current incentive structure with a rebate-only mechanism is a reasonable policy alternative that leads to a negligible welfare loss compared to the joint mechanism which calls for rebates to consumers and additional taxes on GM (such a joint mechanism is difficult to implement politically and may lead to moral hazard, see §6). Second, in the sensitivity analysis section we examine the impact of uncertainty and externality. Our results reinforce the validity of the simplified mechanism with rebates only and suggest that a subsidy-only
mechanism may be very inefficient. This happens because the optimal subsidy is often so high that the firm offers a 100% service level, implying that the government absorbs all the risk in the system, which is costly. We finally show that consumer surplus may increase in uncertainty, i.e., consumers may benefit from demand uncertainty.

In the remainder of the paper, §2 discusses relevant literature, §3 presents our model, §4 discusses the benchmark cases of centralized and decentralized systems, and §5 analyzes the government intervention mechanisms. Numerical analysis of the Chevy Volt example are presented in §6, followed by conclusions in §7. All proofs are in the online appendix.

2 Literature Review

Because our paper considers welfare, externalities, and regulation in a price-setting newsvendor context, it is related to the literature in both operations management and economics.

Within the operations literature, there are two main streams of research related to our paper. The first stream of papers considers social welfare in the area of vaccines, e.g., Chick et al. (2008), Mamani et al. (2011), Adida et al. (2011), Arifoglu et al. (2011), Taylor and Xiao (2013). There are three main differences between those papers and ours. First, those papers are based on infectious disease models, such as SIR, and are thus not readily applicable to products other than vaccines. Second, in most of those papers supply equals demand, while we explicitly consider a loss in welfare due to possible stockouts. Third, those papers typically consider only a small set of government intervention tools, while our approach considers multiple intervention mechanisms in one parsimonious framework.

The second stream of research in the operations literature considers welfare, externalities, and regulation in relation to environmental sustainability, e.g., Atasu et al. (2009), Atasu and Subramanian (2012), Drake et al. (2012), and Krass et al. (2013). Atasu et al. (2009) and Atasu and Subramanian (2012) consider the use of a tax on the firm (negative subsidy in

\footnote{A number of papers consider public interest goods, e.g., Lodree and Taskin (2008) who discuss inventory policies for emergency supplies, but those works are very different from ours and hence we do not discuss them in detail.}
the language of our paper) in the design of the take-back legislation in the closed-loop supply chain context. Drake et al. (2012) consider the use of tax and cap-and-trade mechanisms, and Krass et al. (2013) consider the use of taxes, rebates and fixed cost subsidies (which are different from variable cost subsidies in our paper) in order to motivate a firm to adopt green emissions-reducing technologies. These papers either consider deterministic demand or a situation when demand uncertainty is resolved before price is decided, and thus are quite different from ours.

In relation to our application based on the Chevy Volt example, Huang et al. (2012) discuss how consumers’ bargaining power moderates the government rebates (which they call subsidy to consumers) for EVs. Their follow-up research note Luo et al. (2013) discusses a different kind of the intervention mechanism: price discount with a ceiling that is somewhat similar to our rebates; they do not consider subsidies (as are defined in this paper) at all. Cohen et al. (2013) study the design of the subsidy scheme for green technology adoption with Volt as an example, but instead of the welfare optimization problem that we study, they consider a problem of meeting an exogenously given sales target – an approach that is qualitatively similar to quantity coordination in our paper; Alizamir et al. (2012) also consider a similar objective.

In the economics literature, there is a wide body of work on regulating a monopoly (e.g., see Train 1991 and Berg and Tschirhart 1988 for reviews), but only a small fraction of that literature considers a situation where supply may not equal demand, which is a critical element of the newsvendor-like situation. Most related to ours are the works on peak-load pricing and capacity investments by a power utility under stochastic demand, see Crew et al. (1995) for a review. As we discuss in detail in §3, the major difference between those works and ours is in modeling stockouts. A utility company has a direct control over which customers to deny service to and thus “cuts out” customers with the lowest valuations, while a newsvendor operates on the first-come first-served basis and serves all customers until it runs out of stock regardless of (and without knowing) their valuations.
3 Model

Consider a system that consists of a single newsvendor firm that produces a public interest good, consumers who purchase this good and obtain direct benefit from its consumption, a general population that obtains indirect externality benefit from the production and consumption of the good, and a government that can intervene in the market in order to increase the social welfare.

The newsvendor nature of the good implies that the order quantity, $q$, and the retail price, $p$, must be set prior to observing the demand for the product. Note that while to be consistent with the newsvendor literature we refer to $q$ as “quantity”, as explained in the Introduction, in the Chevy Volt case $q$ corresponds to the production capacity. The firm faces stochastic additive\(^6\) demand, $D(p) = y(p) + \epsilon$, where $\epsilon \in [-B, B]$ is a random variable with cdf $F(\cdot)$ and pdf $f(\cdot)$ such that $E[\epsilon] = 0$, and $y(p) = a - p$ for $p \in [0, a - B]$, where $a > B$. With these assumptions, the social welfare consists of the following four components:

\[ \text{Social Welfare} = \text{Firm’s Profit} + \text{Consumer Surplus} + \text{Externality Benefit} - \text{Govt. Cost} \]

In what follows we formulate the model for each component of the social welfare:

- **Firm’s Profit** is modeled using standard definitions, e.g., Pertuzzi and Dada (1999).

  The firm’s profit for an order quantity $q$, unit cost $c$, retail price $p$ and salvage value $v$ is equal to:

  \[
  \Pi_F(q, p) = p \min[D(p), q] - cq + v(q - D(p))^+ = \begin{cases} 
  pD(p) - cq + v(q - D(p)), & \text{if } D(p) \leq q; \\
  (p - c)q, & \text{if } D(p) > q.
  \end{cases}
  \]

To avoid trivial cases with no sales we assume that $c \in [B, a - B]$.

\(^6\)Another common form of demand uncertainty considered in the Operations Management literature is multiplicative uncertainty, e.g., $D(p) = y(p) \times \epsilon$, where $E[\epsilon] = 1$. Demand functions with the multiplicative uncertainty, however, fail to satisfy one of the basic axioms needed for welfare analysis, the so-called Slutsky symmetry conditions, making welfare analysis with multiplicative demand model “infeasible” (Krishnan 2010). We therefore do not consider the case with multiplicative demand uncertainty in this paper.
Consumer Surplus is generally defined in the economics literature as the area under the demand curve above the given price. In the case where supply may not equal demand, modeling of consumer surplus critically depends on how the unmet demand is rationed in the event of a stockout, i.e., when $D(p) > q$. Brown and Johnson (1969) in their foundational paper (on power utility’s capacity investment under uncertain demand) assumed that the $D(p) - q$ consumers with lowest willingness-to-pay are denied service, reflecting the ability of a utility company to “cut” whatever consumers it wishes in the event of a power shortage; see Figure 1 (a). Visscher (1973) extended their model to several other types of rationing that are based on the willingness-to-pay. Although plausible for a power utility, these assumptions are unreasonable for a newsvendor: if the newsvendor knew how much each consumer is willing to pay then it would not charge the constant price $p$ to all consumers, rather it would charge each consumer her willingness-to-pay and the consumer surplus would be zero.

An assumption that is more consistent with the newsvendor logic is the “first-come first-served”, i.e., consumers arrive in a random order and each consumer could suffer a welfare loss if she happens to arrive when all stock is already sold. Consider the consumer with the highest valuation. Intuitively, such a consumer wants the product

Figure 1: Illustration for the derivation of consumer surplus: in (a) with the standard “economics” assumption, in (b) with the first-come, first-served assumption.
the most and thus suffers the most if she does not get it. The surplus from purchasing
the good for such a consumer is \( a + \epsilon - p \) which by construction equals \( D(p) \). The
probability to find the product unavailable when \( q \) units were produced while the
demand was \( D(p) \) is: \( 1 - q/D(p) \). Thus the loss in welfare due to a possible stockout
for such a consumer is \( D(p) \times (1 - q/D(p)) = D(p) - q \). Extending this logic to all
consumers (and noting that a linear demand curve implies uniform valuations) implies
that the loss is linearly decreasing to zero, so that the consumer with willingness to
pay exactly \( p \) is incurring no welfare loss in the event of stockout (her surplus is zero,
thus such a consumer is indifferent between purchasing or not). See Figure 1 (b) for
an illustration of this case, and note that by construction \( p + D(p) = a + \epsilon \).

This implies that consumer surplus, \( CS(q, p) \) can be calculated as follows:

\[
CS(q, p) = \begin{cases} 
  \frac{(a-p+\epsilon)D(p)}{2} = \frac{D(p)^2}{2}, & \text{if } D(p) \leq q; \\
  \frac{(a-p+\epsilon)q}{2} = \frac{D(p)q}{2}, & \text{if } D(p) > q.
\end{cases}
\]

(2)

- **Externality Benefit** reflects the indirect benefit (or cost, if negative) that production
  and consumption of the good imposes on the economic agents who may or may not
  be involved in production and consumption of the good per se. A common way of
  modeling externality is to assume a constant marginal externality, \( \alpha \), implying that
  the externality benefit equals \( \alpha \times \min[D(p), q] \). Such an assumption is embedded in
  many practical studies by various governments, World Bank, WHO (Boulier et al.
  2007), as well as in theoretical works, e.g., Taylor and Xiao (2013). It also holds for
  the environmental externality that our application considers: using an EV instead of
  a regular vehicle is decreasing air pollution proportionally to the number of EVs.

- **Government Cost** accounts for the government expenditure associated with partic-
  ular intervention tools; this cost should be subtracted to avoid double-counting. As
  per the discussion in the introduction, we consider the following two tools:
<table>
<thead>
<tr>
<th>Tool</th>
<th>Firm’s price, cost</th>
<th>Consumer price</th>
<th>Units action applies to</th>
<th>Government cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebate, r</td>
<td>$p, c$</td>
<td>$p - r$</td>
<td>$\min[q, D(p - r)]$</td>
<td>$r \min[q, D(p - r)]$</td>
</tr>
<tr>
<td>Subsidy, s</td>
<td>$p, c - s$</td>
<td>$p$</td>
<td>$q$</td>
<td>$sq$</td>
</tr>
</tbody>
</table>

In what follows we will discuss the ability and effectiveness of using these tools to improve social welfare. We consider three mechanisms: the joint mechanism in which the government uses both rebates and subsidies simultaneously, and two simplified mechanisms in which the government uses only rebates or only subsidies. To do so we first consider two benchmark cases: the centralized case, where the government decides on the price and quantity to maximize the social welfare, and the decentralized case where the firm decides on the price and quantity to maximize its own profit.

4 Centralized and Decentralized Solutions

Next we apply the social welfare model to the benchmark cases of centralized (first-best) and decentralized (unregulated) systems. Note that the government cost in either case is zero.

4.1 Centralized (First Best) Case: Optimizing Social Welfare

In this case, the firm’s production quantity and price are set to maximize social welfare. Let $SW_C$ denote the social welfare; subscript “$C$” denotes the centralized case.

To facilitate the analysis (similar to Petruzzi and Dada, 1999) we use a change of variables and let $z = q - y(p)$. Hence we use $z$, the stocking factor, instead of $q$ in our analysis. Using the model developed in §3, the social welfare becomes:

$$SW_C(z, p) = \begin{cases} 
(p + \alpha)(y(p) + \epsilon) - c(y(p) + z) + \frac{(y(p) + \epsilon)^2}{2} + v(z - \epsilon), & \text{if } \epsilon \leq z; \\
(p + \alpha)(y(p) + z) - c(y(p) + z) + \frac{(y(p) + z)^2}{2}, & \text{if } \epsilon > z.
\end{cases}$$

(3)

Taking expectation over $\epsilon$, and using the fact that $\int_{-B}^{z} \omega f(\omega) d\omega = -\int_{z}^{B} \omega f(\omega) d\omega$ because
$E[e] = 0$, the expected social welfare becomes:

$$E[SW_C(z, p)] = \left(\frac{y(p)}{2} + p + \alpha\right)(y(p) - \Theta(z)) + \frac{1}{2} \int_{-B}^{B} \omega(\omega - z)f(\omega)d\omega - c(y(p) + z) + v(\Theta(z) + z)$$

(4)

where $\Theta(z)$ captures the expected shortages and is defined as follows:

$$\Theta(z) \equiv \int_{z}^{B} (\omega - z)f(\omega)d\omega > 0$$

(5)

We have the following result (all proofs are in the online appendix):

**Lemma 1** If $F(z)$ has increased failure rate (IFR$^7$), $f(-B) > \frac{1}{2(a+c+\alpha-v)-3B}$, $\alpha > 2v + B - a - c$, and $\alpha < c - B/2$ then the optimal retail price and stocking factor are given by:

$$p_C^* = c - \alpha - \frac{\Theta(z_C^*)}{2} \geq 0;$$

$$\tilde{F}(z_C^*) = \frac{2(c-v)+\int_{-B}^{z_C^*} \omega f(\omega)d\omega}{a+2(\alpha-v)+p_C^*} = \frac{2(c-v)-\Theta(z_C^*)}{(a+c+\alpha-v)+z_C^* + \frac{\Theta(z_C^*)}{2}}.$$  

(6)

Similar to Petruzzi and Dada (1999) the second equation in (6) should be used when it has an interior solution on the $[-B,B]$ interval; otherwise the solution is on the boundary$^8$.

The price and stocking factor in Lemma 1 are socially optimal in the sense of the first-best solution: they maximize the expected welfare. Note that without externality and demand uncertainty, Lemma 1 reduces to the problem with only one variable (price) and the standard economics result that the centralized optimal price equals marginal cost (e.g., see Chapter 5 in Mas-Colell et al. 1995). The case with externality but without uncertainty is also well-studied in economics: the central planner wants to increase sales of goods with positive externality and thus selects price below marginal cost, and wants to decrease sales of goods with negative externality hence pricing them above cost (e.g, see Chapter 11 in Mas-Colell

$^7$The IFR assumption is common in the analysis of newsvendor models and holds for the uniform, normal and other commonly used distributions. The condition on the density function in this and following Lemmas is a technical sufficient condition to guarantee optimality. The condition can be easily checked for any distribution and it holds when $a \gg B$ which encompasses most practical situations. If the condition does not hold, then one needs to perform a complete search over $z$, similar to the approach used by Petruzzi and Dada (1999).

$^8$For example, if $F$ is uniform, then the second equation is a cubic equation, which generally had three roots, but at most one of them is on the $[-B,B]$ interval.
et al. 1995). The first two conditions in the Lemma are technical, while the last one ensures that the centralized price is non-negative\(^9\), which we believe is a reasonable assumption for most products, the EV category in particular.

The novelty of our result is that we consider both externality and uncertainty and the complex interaction of the two (resulting in a problem with two variables: price and stocking factor\(^{10}\)). As Lemma 1 shows, even when \(\alpha = 0\), demand uncertainty lowers consumer surplus (relative to the case with no demand uncertainty), and thus the central planner would prefer to avoid stockouts, resulting in a price that is lower than the marginal cost. Note that the condition \(\alpha > 2v + B - a - c\) holds for a positive \(\alpha\) automatically because \(2v + B - a - c = 2(v - c) - (a - c - B)\) and \(c > v, c < a - B\) by definition. For goods with positive externality this effect reinforces that of the externality, resulting in an even lower price, but for the goods with negative externality the two effects are in conflict: when externality is negative and large the price can be above cost, but when it is negative and small the price could be above or below depending on other model parameters. The Corollary below formalizes this discussion:

**Corollary 1** If \(\alpha \geq 0\) then the centralized price is below the marginal cost; if \(\alpha \leq -\frac{B}{2}\) then the centralized price is above the marginal cost, otherwise the comparison is inconclusive.

### 4.2 Decentralized Case

The result for the decentralized case follows from Petruzzi and Dada (1999), but we nevertheless present it to facilitate comparisons:

**Lemma 2** (Equivalent to Lemma 1 and Theorem 1 in Petruzzi and Dada 1999) If \(F(z)\) is

\(^9\)For certain rather specific products it might make sense for the government to pay consumers for “buying” the product, i.e., have a negative price. For example, due to Ebola threats in 2014 a negative price for an Ebola vaccine may be reasonable because the externality cost for other people from someone who may have the disease is very high. This is precisely what the condition states: if \(\alpha > c - B/2\) then the centralized price can be negative.

\(^{10}\)A fundamental difference between our approach and the economics results mentioned above is that they consider public goods for which, by definition the concept of “stocking factor” does not apply. We consider public interest goods, i.e., a setting when a certain quantity of commercial good with externality is produced.
IFR, and \( f(-B) > \frac{1}{(a+c-v-B)} \), then the optimal retail price and stocking factor are given by:

\[
\begin{align*}
\begin{cases} 
p_D^* = \frac{a+c}{2} - \frac{\Theta(z_D^*)}{2} \geq 0; \\
F(z_D^*) = \frac{c-v}{p_D^*} = \frac{2(c-v)}{(a+c)-\Theta(z_D^*)}.
\end{cases}
\end{align*}
\]

Observe that as in the centralized case, demand uncertainty lowers the decentralized price, but since \( \Theta(z) \leq B \) while by assumption \( c \geq B \), the decentralized price is always non-negative, which is natural as otherwise the firm will not participate in the market.

4.3 Comparison Between Centralized and Decentralized Cases

Fundamentally the difference between the centralized and decentralized solutions is due to two factors. First, the firm in its profit maximization decision does not internalize the externality that the centralized planner takes into account, and second, the firm maximizes its profit, while not taking the consumer surplus into account. Since this paper is about public interest goods, we focus on the impact of externality:

**Proposition 1** Comparing the centralized and decentralized cases:

a) For goods with positive externality \( (\alpha \geq 0) \) and low salvage value \( (v \leq \alpha/2 \text{ is a sufficient condition}) \), the optimal stocking factor in the centralized case is higher and the price is lower than in the decentralized case. However due to the demand uncertainty, the decrease in the service level mitigates a portion of the increase in price.

b) For goods with large negative externality \( (\alpha \leq \min[-(\frac{a-c}{2} + B), -(a + c) + 3B/2] \text{ is a sufficient condition}) \) and zero salvage value, the optimal stocking factor is lower and the price is higher in the centralized case.

c) Otherwise, i.e., for goods with small negative externality or large salvage value, the comparison is inconclusive: the centralized price and stocking factor could be either higher or lower depending on the other model parameters.
The intuition for this result is in the interplay between the externality and other components of the welfare. When externality is positive, it is in society’s interest to produce/consume more of the good – consumers receive private benefit (utility) from consuming the good themselves, and receive externality benefit when others consume the product – hence the centralized price will be low and quantity high. When externality is negative, the components of the welfare are in conflict: consumers receive private benefit from consuming the product themselves, but receive a disutility when others consume the product (negative externality). When externality is large and negative the externality component outweighs the combined consumer+firm surplus, implying that it is in the society’s best interest that little of the good is produced and consumed, therefore the centralized price will be high and quantity low. When externality is negative but small, however, it is not clear which of the two components will dominate. The decentralized solution will be identical in all of these cases since the firm is only maximizing its profit regardless of externality: the expressions in (7) are independent of $\alpha$. Thus for parts (a) and (b) there is a clear-cut ordering\footnote{We note that without demand uncertainty, following the standard economics results (e.g., Chapter 11 in Mas-Colell et al. 1995), (b) and (c) would combine and the ordering between (a) and (b-c) would follow.} between the centralized and decentralized prices and service levels, while for part (c) there is none.

5 Analysis of Government Coordination Mechanisms

We distinguish between the rebates and subsidies tools themselves, and the mechanisms in which they are used. Specifically, we first consider the joint (or combined) mechanism in the form of $(r, s)$, where $r$ is the rebate given to consumers, and $s$ is the cost subsidy given to the firm\footnote{Working on this paper we have considered a buy-back tool (Lariviere 1999), but found it very inefficient; we therefore do not present results for buy-backs.}. We use subscript $J$ to denote such a joint mechanism. We then discuss two simplified mechanisms in the form of $(r, 0)$ and $(0, s)$ which correspond to the case when the government only uses one of the two tools. We note that we do not restrict $r, s$ to be positive: $r, s > 0$ correspond to rebates and subsidies in their typical interpretation, and
\( r, s < 0 \) correspond to taxes paid either by the consumers (e.g., sales tax) or the firm (e.g., environmental tax). As we show next, this flexibility of allowing \( r, s \) to be negative has an important implication for coordination.

### 5.1 Firm’s best response to \((r, s)\)

The firm’s expected profit in the case with the joint mechanism \((r, s)\) equals:

\[
E_{r,s}[\Pi(z, p) |_{r,s}] = p(y(p - r) - \Theta(z)) - (c - s)(y(p - r) + z) + v(\Theta(z) + z) \tag{8}
\]

Maximizing the expected profit in (8) we obtain the firm’s optimal price and stocking factor in response to \((r, s)\):

**Lemma 3** If \( F(z) \) is IFR, and \( f(-B) > \frac{1}{(a+c-v-s+r-B)} \), then the firm’s optimal retail price and stocking factor in response to policy \((r, s)\) are given by:

\[
\begin{align*}
\bar{p}^*(r, s) &= \frac{a+c-s+r}{2} - \frac{\Theta(z^*(r,s))}{2}; \\
\bar{F}(z^*(r, s)) &= \frac{c-r-s}{\bar{p}^*(r,s)} = \frac{2(c-v-s)}{(a+c-s+r) - \Theta(z^*(r,s))}
\end{align*}
\tag{9}
\]

where the price that consumers pay equals, \( p^*(r, s) - r = \frac{a+c-s+r}{2} - \frac{\Theta(z^*(r,s))}{2} \).

Note that the optimal price in (9) could be negative for some values of \( s \) and \( r \). However, in order to maximize welfare the government will be selecting \( s \) and \( r \) such that the price in (9) equals the centralized price (which as per Lemma 1 is non-negative). Thus for the optimal \( s \) and \( r \) the price in (9) will never be negative.

It is interesting to note that in Lemmas 1–3, the structure of the corresponding optimal price is very similar: a deterministic component minus half the expected shortages. Since the expected shortages are non-negative, the resultant optimal (stochastic) price is below the deterministic component (which is the price under deterministic demand). For the decentralized case and additive demand this relationship between the stochastic and deterministic prices was first shown by Mills (1959); for more general demand models, however, the relationship may be different, see Petruzzi and Dada (1999) and Raz and Porteus (2006).
Lemmas 1–3 also have an interesting implication on how uncertainty impacts consumer surplus. From (5), for a given $z$, $\Theta(z)$ is increasing in $B$, i.e., the expected shortages increase in demand uncertainty resulting in a lower price and a higher consumer surplus. The optimal $z^*$, however, will itself change with uncertainty, but even if the optimal quantity decreases (if $z^*$ decreases significantly), the decrease in price could offset the decrease in quantity and the consumer surplus could as a result be increasing in demand uncertainty. That is, consumers could benefit from demand uncertainty. We note that Cohen et al. (2013) also establish the same qualitative result, which in their setup they show analytically. In our setup this result is not guaranteed to always hold analytically, but in the numerical study of Chevy Volt in §6, consumers will indeed benefit from demand uncertainty. Next we use the firm’s best response to analyze the joint and simplified mechanisms.

5.2 Joint mechanism $(r, s)$.

To derive the coordinating rebate and subsidy we solve for the $(r^*, s^*)$ that equate the firm’s best response consumer price and stocking factor given above to those of the centralized case in equation (6). We have the following result:

**Proposition 2** There exists a unique joint mechanism that achieves the centralized social welfare (i.e., coordinates the system). The coordinating values of rebate and subsidy are:

$$
\begin{align*}
    r_J^* &= (a - c) + 2\alpha - s_J^*; \\
    s_J^* &= \frac{(a-a-\Theta(z_C^*))((c-v)+(c-v)(z_C^*-c))}{(a+a+\Theta(z_C^*))+(z_C^*-c)}
\end{align*}
$$

An important observation from Proposition 2 is that while the optimal joint subsidy is a non-intuitive function of the model parameters, the optimal joint rebate shows a clear relationship to the subsidy. An increase in the optimal subsidy given to the firm results in a decrease in the optimal rebate given to consumers. This also implies that if a subsidy is large enough, then the optimal rebate can be negative, and vice versa, a fact that could make the joint mechanism hard to implement, e.g., for political reasons.
It is also easy to see that generally $r^*_j \neq 0$ and $S^*_j \neq 0$, and thus the simplified policies do not achieve the centralized welfare (i.e., coordinate both price and quantity). But as we discuss next, they both can coordinate either the price or the quantity, which as we will see in the numerical study, often achieves welfare that is very close to the centralized optimum.

5.3 Simplified mechanisms $(r, 0)$ and $(0, s)$

Given that the joint mechanism may be hard to implement, while the simplified mechanisms cannot coordinate welfare, next we consider two alternative coordination goals: coordinating price or coordinating quantity. Such goals are consistent with practice: for example, the Obama administration’s goal of putting one million EVs on the road by 2015 (DOE 2011) is identical to coordinating quantity. The goal of quantity coordination is also used as the regulatory objective in Alizamir et al. (2012) and Cohen et al. (2013).

Equating the firm’s best response price $p^*(r, 0)$ from Lemma 3 to the centralized optimal price from Lemma 1 and solving $r$ gives the value of rebate that coordinates price. We refer to such a value as $r^*_{\text{price}}$. Repeating similar procedure for mechanism $(0, s)$, as well as for the goal of coordinating quantity we obtain the following proposition:

**Proposition 3** The simplified mechanisms can coordinate price/quantity as follows:

a) There exists a value of rebate $r^*_i, i = \{\text{price, quantity}\}$, such that the simplified mechanism $(r^*_{\text{price}}, 0)$ coordinates the retail price and $(r^*_i, 0)$ coordinates the quantity.

b) There exists a value of subsidy $s^*_i, where i = \{\text{price, quantity}\}$, such that the simplified mechanism $(0, s^*_{\text{price}})$ coordinates the retail price, and $(0, s^*_i)$ coordinates the quantity. Further, there exists $\hat{c}_i$ that solves $\hat{c}_i - s^*_i(\hat{c}_i) = v_i$, where $v_{\text{price}} = 0, v_{\text{quantity}} = v$, such that if $c \leq \hat{c}_i$ then $F(z^*_i) = 0$.

By Proposition 3 the rebate-only and subsidy-only mechanisms can coordinate either price or quantity. However, an important observation from Proposition 3(b) is that when
the product cost is small, the subsidy-only mechanism results in 100% service level. As we will show in §6, this leads to large welfare losses under the subsidy-only mechanism.

Note that, technically speaking, one can consider the two simplified mechanisms with the goal of maximizing welfare, possibly coordinating neither price nor quantity. We examined this goal and found that the total welfare achieved of the simplified mechanisms under this goal is not that different from that under the goals of coordinating price or quantity, and therefore do not include such analysis in the paper.

5.4 Properties of the coordination mechanisms

We next outline a fundamental property that contrasts the joint and simplified mechanisms:

**Proposition 4** The following relationships hold for the optimal rebates and subsidies:

a) If $\alpha \geq \frac{(c-v)(B+c)}{(c-v)-B} - a$ and $B \leq (c-v)$, then under the joint mechanism $s_J^* \leq 0$. If in addition $\alpha \geq \frac{c-a}{2}$, then under the joint mechanism $r_J^* \geq 0$,

b) If $\alpha \geq \frac{c-a}{2} + \frac{3B}{2}$, then under either of the simplified mechanisms $s_i^* \geq 0$ and $r_i^* \geq 0$, where $i = \{\text{price, quantity}\}$.

Proposition 4 has a number of profound implications. First and foremost, part (a) states that when the product’s externality is larger than a certain threshold, the coordinating “subsidy” under the joint mechanism is in fact a tax. The condition for part (a) holds when the market size ($a$) is large, the cost ($c$) is small, the demand uncertainty ($B$) is small and the externality ($\alpha$) is large enough (positive or negative but small). In that case the government finds it more beneficial to coordinate via consumers: by providing a rebate it rewards consumers for purchasing the good and inflates the firm’s demand, hence increasing production. The government then taxes the firm with a negative subsidy for each unit of production. But when the condition does not hold, the government coordinates via the firm, subsidizing the firm and taxing consumers. Large negative externality makes it harder
Figure 2: Optimal joint subsidy (left) and rebate (right) as a function of uncertainty ($B$) and externality ($\alpha$) for $\epsilon = Uniform[-B, B]$, $c = 3$, $v = 1$ and $a = 5$.

to satisfy the condition as does high uncertainty, high cost or small market size. Positive externality, low cost or large market size make the condition easier to satisfy.

Figure 2 illustrates this discussion for $c = 3$, $v = 1$ and $a = 5$. When externality is larger than approximately $-1$, the optimal subsidy under the joint mechanism is negative and the rebate is positive. Only when externality is very small and demand uncertainty is very large the conditions in Proposition 4(a) are violated and the joint subsidy becomes positive and rebate becomes negative.

The reason for why the government prefers to coordinate by providing a large rebate and taxing the firm is the following. By providing a rebate the government gives the firm an opportunity to raise the price without losing demand; e.g., it can raise the price by the level of the rebate, so that the price consumer pays does not change. For the firm higher price implies higher margin, in response to which it increases the service level, i.e., achieves a higher profit. For consumers, no change in price but an increase in the service level implies an increase in the surplus and externality. For the government, however, a rebate on its own is very expensive. Adding a tax reduces the government cost. Further, since the rebate applies only to the units sold, while subsidy applies to all units produced (even to those that are not sold) the tax may be smaller in absolute terms than the rebate (because it applies to more units). Thus, the net transfer to the firm (rebate minus tax) can be positive with all the above-mentioned benefits for the firm and consumers effectively at no cost to the
government. This logic works in cases when it is beneficial for the society to encourage the firm to produce more, e.g., when the externality is positive and salvage value is small, hence the condition in part (a).

Part (b) of Proposition 4 states that under conceptually\textsuperscript{13} similar condition in the case of either of the simplified mechanisms, both subsidy and rebate are positive; otherwise, when uncertainty is high and externality is low (a sufficient condition is $\alpha \leq \frac{c-a}{2} - \frac{3B}{\pi}$), the simplified mechanisms could result in taxes on the firm or consumers.

6 Rebates and Subsidies for EV Adoption

In this section we use our model to examine rebates and subsidies for electric vehicles (EVs). EVs are a natural example that embodies the object of this paper’s study: public interest is evident, demand uncertainty is present, environmental externality is clear and quantifiable, and government regulation with rebates and subsidies is utilized. We base our study on a specific example: Chevy Volt – an extended range electric vehicle produced by General Motors (GM). As we discuss below, this example fits the monopolistic newsvendor framework of our model reasonably well.

Regarding the monopoly assumption, while GM is certainly not a monopolist in the EV market, Volt’s dominant position (70% market share\textsuperscript{14} in 2012, market leadership\textsuperscript{15} in 2013 and forecasted market share of 50% till 2015, DOE 2011) implies that GM is a market leader that can assume that the other firms’ response is already captured in its demand function. Thus GM can optimize its decisions as if the decisions of those other firms are fixed, hence the monopoly assumption is a reasonable approximation. Given the recent hype about Tesla, which in the public’s perception became the EV, we note that while it indeed sold more

\textsuperscript{13}We say that these conditions are conceptually similar because the model parameters affect them in the same way: larger externality and larger market size make the condition easier to satisfy, while larger cost and larger uncertainty make it harder to satisfy.

\textsuperscript{14}According to http://www.goodcarbadcar.net, in 2012 GM sold 23,461 Volts in the U.S. and 1,225 in Canada, more than twice outpacing the nearest competitor, Nissan Leaf, which sold 9,819 and 240 vehicles, respectively.

\textsuperscript{15}http://cleantechnica.com/2014/01/05/chevy-volt-sales-inch-nissan-leaf-sales-2013/
cars in 2013 than expected, Volt still sold 30% more. More importantly, consumers who consider buying a Volt and are impacted by the government incentive (currently $7,500 in the US) are unlikely to consider purchasing an $80,000 car like Tesla’s Model S, and vice versa. Therefore, Tesla is not really a competitor for the Volt.

Regarding the newsvendor assumption, note that the quantity decision in our example is not how many vehicles to produce, but rather how much capacity to allocate to the manufacturing of Volt at the beginning of the year. Since Volt is quite different from other car models manufactured by GM, it requires dedicated production capacity that is decided in advance and cannot be easily changed during the selling season. If demand exceeds capacity then ramping up production is costly if not impossible\textsuperscript{16}. If demand is below capacity, then one way\textsuperscript{17} to deal with it for an auto manufacturer is to temporarily halt production, as GM in fact did with Volt for 5 weeks in April/May 2012\textsuperscript{18}. Doing so, however, is also costly because production involves both variable costs (raw materials, energy, labor) and fixed costs (security/heating/cooling, property taxes, finance charges, management fees). While the latter costs are fixed in the sense that they do not depend on the number of vehicles assembled, they depend on the capacity decision, e.g., a production line with larger capacity would be more expensive and hence have a higher finance charge. In the language of the newsvendor model, given the capacity decision, the total cost per unit (variable plus total fixed divided by capacity) would be considered as “cost”, while the variable cost could be framed as “salvage”: indeed, planning to produce a vehicle but not doing it (i.e., incurring only fixed costs) is equivalent to producing it first and then salvaging the variable costs back; see an example on pages 353-4 in McGuigan et al. (2008). In other words, while EVs themselves are not perishable and have multiple replenishment possibilities, the capacity to manufacture Volt – the decision in our model – is perishable and has limited replenishment,

\textsuperscript{16}For example, BMW, clearly a very capable auto manufacturer, was unable to ramp production of its Z3 model when demand following the model’s appearance in the 1995 James Bond movie exceeded the production capacity (Fisher 1997, Fournier and Dolan 1997). We note that at that moment Z3 was also very different from other models produced by BMW, much as Volt is for GM now.

\textsuperscript{17}Another way is to offer end-of-the-year discounts, which can also be interpreted as a salvage value.

\textsuperscript{18}http://money.cnn.com/2012/03/02/autos/volt_production_stopped/
therefore our example fits with the newsvendor model well.

With this, we present two sets of analyses for the Volt example: first we consider the base-case scenario (using the present-day estimates for the model parameters), and second we discuss sensitivity of our results with respect to uncertainty and externality.

6.1 Parameter Estimation

Demand and cost parameters, \( a, c \). Volt’s price is approximately USD40,000\(^{19}\), and the average margin on a new car is 14.4\%\(^{20}\), which suggests that the cost of a Volt is approximately USD34,000. We will thus assume \( c = 34 \) (in thousands). In 2012 GM sold\(^{21}\) approximately 25,000 Volts, thus from \( y(p) = a - p \) with \( p = 40 \) and \( y(40) = 25 \) we reverse-engineer \( a = 65 \). Volt buyers in the U.S. are eligible for a tax credit of USD7,500. That is, the government is currently using the rebates-only mechanism \((r, 0) = (7.5, 0)\).

Salvage value parameter, \( v \). An important parameter in our analysis – one that in fact allows us to apply the newsvendor framework to the Chevy Volt example, is the salvage value. As discussed in the Introduction, salvage value corresponds to the variable production cost, so that not producing a Volt is equivalent to producing it first at the cost of \( c = 34 \) as estimated above, and then recovering \( v \) back. There are several sources of data to estimate the split between fixed and variable cost for Volt. Buss (2010) citing Citi Investment Research states that “GM’s fixed costs per vehicle will drop from $13,591 last year [2009] to $8,165 this year [2010] and to $6,726 by 2012.” Whether this estimate materialized is not clear, and further the fixed cost of manufacturing Volt is arguably higher because of the dedicated tooling and personnel. Lutz (2012) suggests that manufacturing a Volt involves $10,000 in electric components (battery, motors, wiring, etc.), and additional $1,000 in labor, but otherwise Volt “is about equal to a Chevy Cruze which sells for around $22,000 retail.” With Cruze being much more of an average car than Volt, we could reverse engineer that

\(^{19}\)http://www.chevrolet.com/volt-electric-car.html
\(^{20}\)http://www.nada.org/NR/rdonlyres/C1C58F5A-BE0E-4E1A-9B56-1C3025B5B452/0/NADADATA2012Final.pdf
\(^{21}\)http://www.goodcarbadcar.net
$3,300 \approx 14.4\% \times $22,000$ is GM’s margin, $6,700$ is fixed cost (as per Buss’s projection), which leaves $12,000$ as variable costs. Combining this with the cost of electric components and extra labor, we obtain the variable cost of $23,000$, which agrees with an estimate of $24,000$ from Blanco (2012). That is, we will use $v = 23$ in our analysis.

**CO2 emissions.** We estimate the emissions of an average vehicle in the U.S. to be $10.8 \times 11,489 \times 8.92 \times 10^{-3} / 24 = 46.11$ metric tons of CO2 per vehicle, where $10.8$ is the average lifetime of a vehicle in years$^{22}$, $11,489$ is the average miles traveled per vehicle per year, $8.92 \times 10^{-3}$ is the average CO2 emissions in tons per gallon$^{23}$, and $24$ is the average fuel economy of a vehicle in miles-per-gallon (MPG)$^{24}$. Following the same logic, and taking into account that Chevy Volt’s MPG is $94^{25}$ suggests that a Chevy Volt emits $11.77$ metric tons of CO2 over its lifetime, savings of $34.34$ metric tons of CO2 over an average vehicle.

**Externality parameter, $\alpha$.** An obvious externality benefit from using a Volt as opposed to the average vehicle is in the decrease of CO2 emissions. There may be other benefits such as spill-over R&D, building a network of charging stations, investments in “smart grid” technology that will allow to charge EVs overnight from the base-load electricity which is current wasted, etc. But to be conservative we will perform our analysis based only on the externality from CO2 emissions.

There are two ways to estimate externality from CO2 emissions. One is to consider the so-called social cost of carbon (SCC). Greenstone et al. (2011) estimated SCC to be USD21 per ton (in 2010 dollars) with the suggested range for sensitivity analysis of USD5 - 65. Nordhaus (2011) estimated SCC to be $12$ (in 2005 dollars). An alternative is to consider market prices for CO2. In 2012 the Australian carbon tax was AUD23 (USD24.2)$^{26}$ and in the European Union the price of carbon emission allowance was between 2.8 and 32 Euros.

$^{22}$http://usatoday30.usatoday.com/money/autos/story/2012-01-17/cars-trucks-age-polk/52613102/1
$^{23}$http://www.epa.gov/cleanenergy/energy-resources/refs.html
$^{24}$http://www.umich.edu/~untriswt/EDI_sales-weighted-mpg.html
$^{25}$http://www.fueleconomy.gov/feg/noframes/31618.shtml. Note, this figure includes emissions from electricity generated to charge Volt, and is based on the average driving pattern, i.e., includes the gasoline emission when gas is used to recharge the battery.
$^{26}$http://www.carbontax.net.au/category/what-is-the-carbon-tax/
(USD3.6 - 42, the average of 24.6)\(^{27}\). Taking inflation into account, the SCC and the market estimates are remarkably similar. Thus, for the base-case we will use the average market estimate between Australia and EU of USD24.4. For sensitivity analysis we will use the range from the EU carbon price which we extend two-fold on the upper end (reflecting expectation of an increase in the price of carbon in the future), i.e., USD3.5 - 84. Note that our range includes the suggested SCC sensitivity range from Greenstone et al. (2011). With these estimates, the externality benefit from using a Volt is on average USD24.4 \(\times 34.34 = 838\), with a range of USD123-2,900. Thus in our base-case analysis we will assume \(\alpha = 0.84\) (in thousands) and in the sensitivity analysis we will consider \(\alpha = 0.12\) for the “low” and \(\alpha = 2.9\) for the “high” externality cases.

**Demand uncertainty parameter, \(B\).** The projections for the Chevy Volt demand are divergent, e.g., DOE (2011) forecasted sales of 120,000 in 2012, while average sales turned out at approximately 25,000. We do not believe this is due to shortages, but rather an exaggerated projection. From monthly sales data, the standard deviation of annual sales is approximately 5,000 vehicles. Thus for the base case we will assume \(B = 5\) and for the sensitivity analysis will consider \(B = 1\) for “low” and \(B = 10\) for “high”. We acknowledge that this approach substitutes the ex-ante estimate of demand uncertainty with ex-post variability in sales, however, in the absence of the former we believe the latter is reasonable.

### 6.2 Base-case Results

Table 1 presents the results for the Chevy Volt example. Several observations are in order.

**Observation 1** Illustrating Proposition 4, the joint mechanism results in a positive rebate to consumers and a negative subsidy (tax) on GM. Simplified mechanisms result in positive rebates and positive subsidies.

As noted earlier, implementing the joint mechanism may be difficult for political reasons, and also because of moral hazard: indeed if GM is taxed for producing a product with

\(^{27}\)Data from http://www.pointcarbon.com/, assessed April 10, 2013
positive externality, then it may be potentially incentivized to make the externality smaller in order to decrease the tax. This favors simplified mechanisms, especially if the resultant welfare loss is small.

**Observation 2** The current structure of the incentive mechanism (rebates-only) is a reasonable policy choice: regardless of the coordinating goal (price or quantity), the welfare loss is minimal, 0.5% only. The subsidy-only mechanism does much worse relatively to rebates-only: loss of up to 5%, i.e., nearly 10 times larger loss.

The poor performance of the subsidy mechanism may seem surprising, but it has an explanation. Notice that when coordinating price in Table 1, \(z = 5\). Since \(B = 5\) this implies that the firm offers a 100% service level; this is the case with \(c \geq \hat{c}_{\text{price}}\) from Proposition 3(b). Thus the subsidy over-rewards the firm and benefits consumers, but at a very high cost\(^{28}\) as the government effectively “buys-out” all the risk: consumers have no risk of stockout, and GM receives subsidy for all units planned for, even those it never produces. Rebates apply only to the units sold and therefore are more efficient, resulting in a smaller welfare loss.

**Observation 3** The current rebate (7.5) appears too small compared to the optimal (\(\approx 30\)).

The difference between the optimal and current rebates could come from two sources. First, our demand function, for simplicity, assumes a slope of 1, \(y(p) = a - 1 \times p\), while it may

---

\(^{28}\)Note that as a result of the intervention both the profit and the consumer surplus increase. However, the social welfare is net the government cost, and that cost needs to be allocated to the members of the society, thus decreasing the profit and/or the consumer surplus. The exact amount of the decrease will depend on the chosen allocation, however, our model is oblivious to that choice.
be different, e.g., \( y(p) = a - b \times p \), where \( b \) is the slope. We have no data to estimate what the slope is, and for the theoretical results this is irrelevant because units of measurement can be re-scaled. The 30 vs 7.5 difference implies that the slope might be approximately 4: if that is the case then the current rebate could be optimal, but the slope of 4 seems overly steep. Second, the optimal rebate of 30 implies a certain sales target for the quantity of Volt’s sold. It could be that the current target (one that results from the rebate of 7.5) is too low. Note that the resultant consumer price (≈ 32.8) is very close to the actual consumer price of Volt (32.5 = 40 sticker price minus 7.5 rebate), which further suggests that the potential suboptimality in welfare is due to the quantity produced. Without the data on the slope, however, we cannot differentiate between these two causes, but likely both are contributing to the overall difference.

6.3 Impact of Uncertainty and Externality

Table 2 presents the results for the sensitivity analysis of the Chevy Volt example with respect to the impact of uncertainly and externality with parameters discussed in §6.1.

Observation 4 We observe the following:

(a) The social welfare is decreasing in uncertainty and increasing in externality;

(b) The consumer surplus is always increasing in externality. The consumer surplus for the centralized, decentralized, and price coordination (for both rebates and subsidies) is increasing in uncertainty, while coordinating quantity, it is decreasing in uncertainty.

(c) Rebates perform best with maximum welfare loss of only 1%. Subsidies perform well under low uncertainty and quite badly when uncertainty is high and externality is low.

By part (a), the best case from the social welfare perspective is when uncertainty is lowest and externality is highest, which fits well with standard economic theory. By part (b) consumer surplus is always increasing in externality: the good is simply “better” in the sense
Table 2: Impact of uncertainty and externality for the centralized (C), joint (J), decentralized (D), rebates-only (R) and subsidy-only (S) cases with the goals of coordinating price (P) or quantity (Q).

<table>
<thead>
<tr>
<th></th>
<th>CS ($000)</th>
<th>SW ($000)</th>
<th>SC Loss, %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High Uncertainty</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C, J</td>
<td>495</td>
<td>431</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>130</td>
<td>308</td>
<td>28.5</td>
</tr>
<tr>
<td>R (P, Q)</td>
<td>(510,447)</td>
<td>(427,427)</td>
<td>(0.9,1)</td>
</tr>
<tr>
<td>S (P, Q)</td>
<td>(522,311)</td>
<td>(391,368)</td>
<td>(9.4,14.7)</td>
</tr>
<tr>
<td><strong>Low Uncertainty</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C, J</td>
<td>484</td>
<td>478</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>120</td>
<td>356</td>
<td>25.5</td>
</tr>
<tr>
<td>R (P, Q)</td>
<td>(485,478)</td>
<td>(477,477)</td>
<td>(0.1,0.1)</td>
</tr>
<tr>
<td>S (P, Q)</td>
<td>(487,462)</td>
<td>(473,473)</td>
<td>(0.9,1)</td>
</tr>
</tbody>
</table>

Table 2: Impact of uncertainty and externality for the centralized (C), joint (J), decentralized (D), rebates-only (R) and subsidy-only (S) cases with the goals of coordinating price (P) or quantity (Q).

It is interesting, however, that the consumer surplus may be increasing in uncertainty, i.e., consumers may benefit from more uncertainty in the market. This happens because the decrease in price from uncertainty is larger than the decrease in quantity (see discussion after Lemma 3). Finally, by part (c) rebates always perform better than subsidies which fits our insight from the benchmark case, however, subsidies can perform quite badly when uncertainty is high reaching a nearly 15% welfare loss when coordinating price when externality is low and uncertainty is high. This means that coordination with rebates (regardless of whether the goal is to coordinate only price or only quantity) is a reasonable policy alternative for governments that cannot implement the joint mechanism, but coordination with subsidies might not be.

7 Discussion and Conclusions

This paper, as far as we know, is the first to consider how a government can coordinate the pricing and supply processes for a public interest good that is produced by a price-
setting newsvendor firm. What differentiates such public interest goods from other goods is the effect of externality. We provide a parsimonious framework that allows for positive and negative externality and for positive or negative (interpreted as taxes on consumers or producers) rebates and subsidies.\footnote{While working on this paper we also considered the buy-back tool, but found it performs rather poorly, resulting in only minor improvements in welfare. We therefore did not include the results for buybacks in this paper.}

We consider a system that consists of a newsvendor firm that manufacturers and sells a public interest good, a population of consumers who buy the good, a general population that experiences externality benefit (or cost) and a government that coordinates/regulates the system. The goal of such coordination is not to maximize the profit as is typical in the classical supply chain management literature, but rather to maximize the social welfare, which in our model consists of the newsvendor’s profit, consumers’ surplus, and externality benefit, net the government cost. We start with the benchmark cases of the centralized and decentralized solutions and for most of the paper discuss the properties and performance of the three coordination mechanisms: the joint mechanism that uses rebates and subsidy together, and two simplified ones which use only rebates or only subsidies. Such simplified mechanisms are easier to implement politically and are commonly used in practice, e.g., the US$7,500 tax credit for EVs (the rebate-only mechanism in the language of our paper).

We find that the relationship between the centralized and decentralized solutions depends critically on the sign and magnitude of externality and uncertainty. For goods with positive externalities, the decentralized price is higher than the centralized price (which is also below the cost) and the quantity is lower than the centralized quantity. For goods with large negative externality, the opposite occurs and the centralized price is higher and quantity lower than in the decentralized case. But when the externality is negative yet small, the comparison is inconclusive as it depends on the other model parameters, especially the magnitude of uncertainty, and their impact on the balance between the negative externality and the positive surplus of consumers and the firm.

We analytically show that the joint mechanism coordinates the system (resulting in the
same welfare as the centralized system), but when externality is large enough, the optimal joint subsidy is negative, i.e., it is actually a tax. In those cases the government coordinates the system by providing (positive) rebates to consumers, increasing the firm’s demand and production, and then taxing the firm (with a negative “subsidy”) for this excess production. The situation is opposite when externality is large and negative. The optimal rebate and subsidy (under identical conditions) are generally positive under the simplified mechanisms.

We numerically study the performance of our model based on the example of Chevy Volt. We estimate the model parameters from market and technical data and make a number of observations related to the performance of the joint and simplified mechanisms. Confirming our analytical results we observe a negative subsidy under the joint mechanism, and positive rebates and subsidies under simplified. Very importantly, we find that the rebates-only mechanism performs very well, as measured by the welfare losses (in most cases the average loss is below 1% and worst-case loss is below 3%). This is certainly encouraging, because it suggests that the current structure (rebate-only) of the mechanism to incentivize EV adoption is rather efficient. In contrast, the subsidy-only mechanism performed quite badly, resulting in double-digit maximal losses. The performance of either mechanism decreased in the uncertainty (which is natural and expected), but while the rebates-only mechanism’s performance improved with externality, the performance of the subsidy-only mechanism did not change much. We also observed that consumer surplus increased with uncertainty, i.e.,

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Goal</th>
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<tbody>
<tr>
<td>Joint</td>
<td>Coordinates, but optimal subsidy is often negative, i.e., a tax</td>
</tr>
<tr>
<td>Rebates-only</td>
<td>Cannot coordinate</td>
</tr>
<tr>
<td>Subsidy-only</td>
<td>Cannot coordinate</td>
</tr>
</tbody>
</table>

Table 3: Summary of results.
consumers benefited from uncertainty in the market. Table 3 summarizes our results.

In terms of future research it would be interesting to extend our framework to other goods with externalities. Specifically, while our model of externality describes environmental externality well, in some other cases externality benefit may exist only until a certain fraction of consumers possess the product. For example, the externality benefit can be modeled as \( \alpha \times \min [\min [D(p), q], M] \), where \( M \) represents a cap beyond which externality disappears, see Boulier et al. (2007). While working on this paper we considered such a formulation numerically and observed that all our qualitative insights continue to hold, but a more thorough investigation could be of interest.

References


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Proofs: A Free-Standing Online Appendix

Proof of Lemma 1. Differentiating (4) over $p$, using the fact that $y(p) = a - p$, we obtain that $\frac{\partial}{\partial p} SW_C(z, p) = -p - \frac{1}{2}\Theta(z) + c - \alpha$. Further, since $\frac{\partial^2}{\partial p^2} SW_C(z, p) = -1 < 0$, the expression for the optimal price given $z$ holds.

Similarly, differentiating (4) over $z$, using the fact that $\frac{\partial \Theta(z)}{\partial z} = -\tilde{F}(z)$ and $y(p) = a - p$, we obtain that: $\frac{\partial}{\partial z} SW_C(z, p) = \frac{a + p + 2\alpha - 2v + z}{2} \tilde{F}(z) - \frac{1}{2} f_A z \omega f(\omega) d\omega - (c - v)$. Equating these expressions to zero and rearranging, the expressions for $p^*_C$ and $z^*_C$ in (6) follow.

Taking the second derivative of (4) with respect to $z$: $\frac{\partial^2}{\partial z^2} SW_C(z, p) = -\frac{f(z)}{2}(a + p + 2\alpha - 2v + z) < 0$ because $a + p + 2\alpha - 2v + z = a + c - \alpha - \Theta(z)/2 + 2\alpha - 2v + z \geq a + c + \alpha - 2v - B \geq 0$ since $z - \Theta(z) \geq -B$ by construction and $\alpha > 2v + B - a - c$ by the condition of the Lemma. That is, both second derivatives with respect to $p$ and $z$ are strictly negative.

The determinant of the Hessian is equal to $\frac{\partial^2 SW_C(z, p)}{\partial z^2} \times \frac{\partial^2 SW_C(z, p)}{\partial p^2} - \left( \frac{\partial^2 SW_C(z, p)}{\partial z \partial p} \right)^2 = \frac{f(z)}{2}(a + p + 2\alpha - 2v + z) - \frac{F(z)^2}{4}$, which is strictly positive when $\frac{f(z)}{F(z)} > \frac{F(z)}{2(a + p + 2\alpha - 2v + z)}$. If this condition holds for all $p$ and $z$ then the Hessian is negative definite and so $SW_C$ is concave, which implies that the extremum point in (6) is the unique maximum. Otherwise, observe from (6) that $p^*_C$ as a function of $z$, $p^*_C(z)$, is unique for any $z$. Hence it is sufficient to establish that the Hessian is negative definite for any $z$ along the optimal path of $p^*_C(z)$ -- since the second partial derivatives are strictly negative, when the Hessian is strictly definite there cannot be more than one locally optimal point along this path. Substituting the expression for $p^*_C(z)$ the condition becomes $\frac{f(z)}{F(z)} > \frac{\tilde{F}(z)}{2(a + c + a - v + z - \frac{3\alpha}{2})}$. Note that the right-hand side is decreasing in $z$ because both $\tilde{F}(z)$ and $\Theta(z)$ are decreasing in $z$. Thus when $F(z)$ is IFR, it suffices to satisfy the condition for $z = -B$, which translates to $f(-B) > \frac{1}{2(a + c + a - v - 3\alpha - 3B)}$. The result then follows.

Proof of Lemma 2. The proof follows the same lines of Lemma 1 as well as Lemma 1 and Theorem 1 in Petruzzi and Dada (1999) and is thus omitted for brevity.

Proof of Proposition 1. Comparing the optimal decisions for the centralized and decentralized cases in (6) and (7), it is easy to see that the stochastic part in both the
centralized and decentralized prices are functionally identical, although not necessarily equal because they depend on the optimal stocking factor which would be different for the two cases. It is, however, easy to establish a relationship between the optimal prices: because \( \Theta(\cdot) \in [0, B], a \geq c + B \) it follows that \( p^*_C < c \), and \( p^*_D > c \) and thus, \( p^*_C < c < p^*_D \).

As mentioned earlier, this points to an inherent unattainability of the first-best centralized solution by a firm without government intervention when \( \alpha > 0 \). To compare the optimal stocking factors, observe that:

\[
\frac{2(c - v) + \int_{-B}^{z} \omega f(\omega)d\omega}{a + c + \alpha - 2v - \frac{\Theta(z)}{2}} = \frac{2(c - v) + \int_{-B}^{z} \omega f(\omega)d\omega}{(a + c - \Theta(z)) + (\alpha + \frac{\Theta(z)}{2}) - 2v} < \frac{2(c - v)}{a + c - \Theta(z)} \tag{11}
\]

where the inequality holds because \( \int_{-B}^{z} \omega f(\omega)d\omega \leq 0 \) for all \( z \) (the latter follows from the fact that \( \int_{-B}^{B} \omega f(\omega)d\omega \equiv E[\epsilon] = 0 \) and \( \alpha \geq 0 \), and \( \alpha + \frac{\Theta(z)}{2} - 2v \geq \alpha - 2v \geq 0 \) by the condition of the lemma. The left- and right-hand sides in (11) can be easily recognized as the right-hand sides in the expressions for the optimal stocking factors in the centralized and decentralized cases, respectively. Since \( \bar{F}(z) \) is a non-increasing function, equation (11) therefore implies that \( z^*_C > z^*_D \), and correspondingly \( \bar{F}(z^*_C) < \bar{F}(z^*_D) \) and \( \Theta(z^*_C) < \Theta(z^*_D) \). The interplay of the last two conditions implies that the increase in price, however, is smaller than the increase in the deterministic part (recall \( \Theta_s \) measure the stochastic part of the optimal price).

When \( \alpha \leq 0 \) observe that the condition in (7) is independent of \( \alpha \), while with \( v = 0 \) the numerator in (6) is necessarily positive. At \( \alpha = 0 \) the denominator is also positive because \( a + c + \alpha + z - \Theta(z)/2 > a + c - 3B/2 > 0 \) because \( a, c > B \). Thus when \( \alpha \) is decreasing down from zero, the denominator decreases, causing the left-hand-side in (6) to increase, which implies that \( z^*_C \) is decreasing, and since \( z^*_D \) is independent of \( \alpha \), at some point \( z^*_C \) will be below \( z^*_D \). Specifically, the denominator equals zero at \( \alpha = -(a + c) + 3B/2 \), thus at this or lower value \( z^*_C = -B \leq z^*_D \), hence the condition of the lemma follows.

For part (c) observe that the above logic does not necessary hold if \( v \approx c \) or \( \alpha > -(a + c) + 3B/2 \) since the numerator in (6) may also be negative but the denominator is positive. Thus the comparison is inconclusive.

\[\blacksquare\]
Proof of Lemma 3. The proof follows the same lines as Lemma 1 and is thus omitted.

Proof of Proposition 2. Equating the expression for the optimal consumer price under the joint mechanism in (9) with the expression for the optimal centralized price in (6) and rearranging (noting that when the joint mechanism coordinates the solutions \( z^*_J = z^*_C \)) we obtain the expression for the optimal rebate (as a function of the optimal subsidy). Equating the corresponding expressions for the optimal stocking factor, substituting the optimal rebate function, and solving for \( s \), the expression for the optimal subsidy follows.

Proof of Proposition 3.
The ability of the simplified mechanisms to coordinate the price is as follows:

a) The \((r, 0)\) mechanism can coordinate the retail price, and the optimal rebate in this case is:
\[
r_{\text{price}}^* = (a - c + 2\alpha) + (\Theta(z_C^*) - \Theta(z_{R,\text{price}}^*))
\]
where \( z_{R,\text{price}}^* \) is the solution to \( \bar{F}(z_{R,\text{price}}^*) = \frac{c}{p_C + r_{\text{price}}^*} \) (when exists, or at the boundary otherwise) and \( z_C^* \) and \( p_C^* \) are given in (6). It is not difficult to show that if \( c \in [a + \alpha + 3B/2; 2v] \) then the interior solution is guaranteed to exist.

b) The \((0, s)\) mechanism can coordinate the retail price, and the optimal subsidy in this case is:
\[
s_{\text{price}}^* = (a - c + 2\alpha) + (\Theta(z_C^*) - \Theta(z_{S,\text{price}}^*))
\]
where \( z_{S,\text{price}}^* \) is the solution to \( \bar{F}(z_{S,\text{price}}^*) = \frac{c - s_{\text{price}}^*}{p_C} \) if \( c \geq \hat{c}_{\text{price}} \) and to \( \bar{F}(z_{S,\text{price}}^*) = 0 \) otherwise, \( z_C^* \) and \( p_C^* \) are given in (6) and \( \hat{c}_{\text{price}} \) is the solution to \( c - s_{\text{price}}^*(c) = 0 \).

The result follows by equating the optimal prices in (6) with the best-response price in (9) [note: consumer’s price is decreased by \( r \) when rebates are provided] and solving for the resultant \( r^* \) and \( s^* \). For the latter, since \( \bar{F}(z) = \frac{c - s_{\text{price}}^*}{p_C} \), we have two cases: For low values of \( c \), when \( c < s_{\text{price}}^* \) (i.e., when \( c < \hat{c}_{\text{price}} \)) we get that the right-hand side is negative and thus \( z_{S,\text{price}}^* = B \) (equivalently \( F(z_{S,\text{price}}^*) = 1 \)) while when \( c \geq s_{\text{price}}^* \), the above equation holds.
The ability of the simplified mechanisms to coordinate the quantity is as follows:

**a)** The $(r, 0)$ policy can coordinate the quantity, and the optimal rebate in this case is:

\[ r_{\text{quantity}}^* = (a - c + 2\alpha) + (\Theta(z_C^*) - \Theta(z_{R,\text{quantity}}^*)) + 2(z_C^* - z_{R,\text{quantity}}^*) \quad (14) \]

where $z_{R,\text{quantity}}^*$ is the solution to $\bar{F}(z_{R,\text{quantity}}^*) = c - v$ if $c \geq \hat{c}_{\text{quantity}}$ and $\bar{F}(z_{R,\text{quantity}}^*) = 0$ otherwise, and the retail price $p_{R,\text{quantity}}^* = p_C^* + r_{R,\text{quantity}}^* - (z_C^* - z_{R,\text{quantity}}^*) - (a - c)$, where $z_C^*$ and $p_C^*$ are given in (6). Similar to the previous case, it is not difficult to show that if $c \in [a + \alpha + 2B + v; 2v - B]$ then the interior solution is guaranteed to exist.

**b)** The $(0, s)$ policy can coordinate the quantity, and the optimal subsidy in this case is:

\[ s_{\text{quantity}}^* = (a - c + 2\alpha) + (\Theta(z_C^*) - \Theta(z_{S,\text{quantity}}^*)) + 2(z_C^* - z_{S,\text{quantity}}^*) \quad (15) \]

where $z_{S,\text{quantity}}^*$ is the solution to $\bar{F}(z_{S,\text{quantity}}^*) = c - v$ if $c \geq \hat{c}_{\text{quantity}}$ and $\bar{F}(z_{S,\text{quantity}}^*) = 0$ otherwise. The retail price $p_{S,\text{quantity}}^* = p_C^* - (z_C^* - z_{S,\text{quantity}}^*)$ where $z_C^*$ and $p_C^*$ are given in (6) and $\hat{c}_{\text{quantity}}$ satisfies $c - s_{\text{quantity}}^*(c) - v = 0$.

By the same logic as in the case of coordinating price, from (6) and (9), the optimal quantity for the centralized case equals:

\[ q_C^* = a - c + \alpha + \frac{\Theta(z_C^*)}{2} + z_C^* \quad (16) \]

and

\[ q_R^* = a - c + r + \frac{\Theta(z_R^*)}{2} + z_R^* \quad (17) \]

\[ q_S^* = a - c + s + \frac{\Theta(z_S^*)}{2} + z_S^* \quad (18) \]

Equating the optimal quantities in (16) and (17) the result follows with respect to $p_R^*$ and $\bar{F}(z_R^*)$. 

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Similarly, equating the optimal quantities in (16) and (18), the result follows with respect to \( p_{S}^{*} \), and for \( \bar{F}(z_{S}^{*}) \) we get that:

\[
\bar{F}(z_{\text{quantity}}^{*}) = \frac{c - v - s_{\text{quantity}}^{*}}{p_{C} - (z_{C}^{*} - z_{S,\text{quantity}}^{*})} \tag{19}
\]

We thus again have two cases: For low values of \( c \), when \( c - v < s_{\text{quantity}}^{*} \) (i.e., when \( c < \hat{c}_{\text{quantity}} \)) we get that the right-hand side is negative and thus \( z_{S,\text{quantity}}^{*} = B \) (equivalently \( F(z_{S,\text{quantity}}^{*}) = 1 \)) while when \( c - v \geq s_{\text{quantity}}^{*} \), the equation (19) holds. ■

**Proof of Proposition 4.** For part (a) consider the signs of the numerator and denominator of the expression for \( s_{p}^{*} \) in (9). For the denominator because \( z \in [-B, B], \Theta \in [0, B] \) we obtain that \( (a + \alpha - \Theta(z_{C}^{*})) + (z_{C}^{*} - c) \geq a + \alpha - 3B/2 - c \), which is positive when \( \alpha \geq c + 3B/2 - a \). Note that this implies \( a + \alpha \geq B/2 \geq 0 \).

For the numerator, which equals \( (a + \alpha - \Theta(z_{C}^{*})) (\Theta(z_{S,price}^{*}) - (c - v)) + (c - v)(z_{C}^{*} + c) \), observe that in the first term the first expression is positive as per the above. The second term in the first expression is negative because \( \Theta \leq B \) and by assumption \( B \leq c - v \). Hence \( (a + \alpha - \Theta(z_{C}^{*})) (\Theta(z_{S,price}^{*}) - (c - v)) + (c - v)(z_{C}^{*} + c) \leq (a + \alpha)(B - (c - v)) + (c - v)(B + c) \), which is negative when \( \alpha \geq (c + B)(c - v - B) + a \). Since the condition for the denominator is subsumed in that for the numerator, the result for subsidies follows. The result for the rebate follows immediately from the expression for \( r_{p}^{*} \) in (9) given the subsidy result. This proves part (a).

For part (b), we follow the same logic and substitute the extreme values for \( z, \Theta \) into the expressions from the proof of Proposition 3. For coordinating price we obtain that \( s_{\text{price}}^{*} = (a - c + 2\alpha) + (\Theta(z_{C}^{*}) - \Theta(z_{S,price}^{*})) \geq (a - c + 2\alpha) + (0 - B) \), which is positive when \( \alpha \geq \frac{c-a}{2} + \frac{B}{2} \), and similarly for rebate. For coordinating quantity the corresponding condition is \( \alpha \geq \frac{c-a}{2} + \frac{3B}{2} \), which subsumes the one for price. The result follows. ■